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Application of Numerical Methods for Regulating Unsteady Flow in Prismatic and Natural Open Channels

Mohsen Ezzeldin, Mohamed.T.Shamaa, Omnia Naguib

Abstract— Routing and operation problems are the most highlight problems which must be explained obviously in unsteady flow in open channels. Both routing and operation problems are basic logical interference methods. The routing approach means using Saint- Venant equations for expecting the results in the downstream of the channel from the known conditions in the upstream. At the other end of the spectrum, the operation technique is the inverse computational method which means using the downstream results to calculate the upstream hydrographs or to interpret the reasons of events happened during wave progression. Saint-Venant equations which are based on the discretization of the Preissmann scheme can be solved using an inverse explicit scheme. The inverse explicit finite difference scheme was applied using three different case studies; once for a trapezoidal section in a single channel, the second for a non-prismatic zone of Al-Mansouria canal between Bahr Tnah canal and Snayet regulator and finally it was applied to a trapezoidal section of a channel diverted into two branches. The procedure is performed from the downstream section by proceeding backward in time followed by space backward. The accuracy of the inverse explicit method was checked by applying the results found by this inverse method as an upstream boundary condition in the routing problem. It was found that the method is stable and gave flow hydrographs in the downstream close to the required demand. The implicit Verwey's Variant of Preissmann scheme, Lax explicit scheme and MacCormack explicit scheme were applied in the routing problem in the first case study. A comparison between the three routing methods is presented.

Key words: Saint-Venant equations, inverse computational problem, Preissmann scheme, Lax diffusive scheme, routing problems, MacCormack scheme, Operation problems, explicit finite difference method.

1 INTRODUCTION

WATER is one of the most important inputs for the economic development. In Egypt, the water problem in the 21st century is due to lack of water and the problem of its management. The problem of water is predicted to increase dramatically, especially after the construction of the Al-Nahda dam as it will lead to a deficit of 9 billion cubic meters of river water, increasing to 16 billion annually with climate change. The future looks scare if the government does not succeed in implementing water resources controlling approach that can match the limited fresh water quantity with the increasing demand. As engineers, it’s our role to shed the light on the management of water according to the different water requirements every year. The requirements of water differ according to the condition of the weather, the type of crop planted as every type needs a specific amount of water and differ also according to supply limitations. It is necessary to meet the requirements of water every year without shortage of water or losses at the required time [15]. This control of water can be easily achieved by placing hydraulic structures along water canals and controlling their operation. Unfortunately, the adjustment of hydraulic structures causes unsteady flow in the canals. The unsteady flow conditions can be defined by a group of semi linear hyperbolic partial differential equations. Mathematical solution of the equations can't be easily used except for some simple cases. Therefore, a lot of numerical techniques for the governing equations can be used for the solution at an enormous number of the study channel points, [6].

2 GOVERNING EQUATIONS

Describing open channel flow means identifying the flow variables using two governing equations; the continuity equation and the momentum or energy equation [4].
The governing equations are expressed as:

$$\frac{\partial A}{\partial t}+\frac{\partial Q}{\partial x}=0 \tag{1}$$

$$\frac{\partial Q}{\partial t}+\frac{\partial}{\partial x}\left(\frac{Q^2}{A}\right)+gA\left(\frac{\partial y}{\partial x}+s_f-s_o\right)=0 \tag{2}$$

Or it can be expressed also in the matrix form as:

$$U_t+F_x+S=0 \tag{3}$$

$$U=\begin{pmatrix}A\\ VA\end{pmatrix},F=\begin{pmatrix}VA\\ V^2A+gAy'\end{pmatrix},S=\begin{pmatrix}0\\ -gA(s_o-s_f)\end{pmatrix} \tag{4}$$

3 NUMERICAL SOLUTION METHODS

One of the most common methods of obtaining approximate solutions of partial differential equations is the method of finite difference. The main idea of this technique is to replace the derivatives in the equation by approximate formulas in the form of differential algebraic equations. Although finite difference techniques are simple to be programed, there are a number of difficulties especially that associated with numerical instabilities. There are two types of finite difference schemes which are named as explicit and implicit finite difference schemes. Each of them has its advantages and drawbacks, so using an implicit scheme or explicit scheme depends on the problem to be solved [1, 3, 12]. The explicit techniques depend mainly on dividing the current domain in x-t plane into small rectangular grids of space Δx and time Δt. The flow variables at a rectangular grid point on a progressive time can be calculated from the known conditions at the present time or present and previous time lines [6, 11]. Although explicit finite difference techniques are relatively simple to be programmed, they are fraught with difficulties associated with instability that go beyond the satisfaction of the courant condition. At the other end of the spectrum, implicit finite difference schemes depend on replacing the spatial partial derivatives and the coefficients in terms of the values at the unknown time level. It is a computational scheme where the values of the parameters like water depth and velocity are determined by solving a system of simultaneous equations using algorithmic methods; e.g. Double sweep method or Newton Raphson method. This means that implicit methods require an extra computation time and more complex process than explicit methods, but there is an obvious advantage of the implicit methods that they are unconditionally stable.

3.1 Lax Diffusive Scheme

The two stage Lax diffusive scheme is second order accurate in space and time technique. By using Lax method, the flow depth, velocity or any other variables can be calculated easily by using the known variables at the previous time stage.
So any flow variables can be approximated as follows [4]:

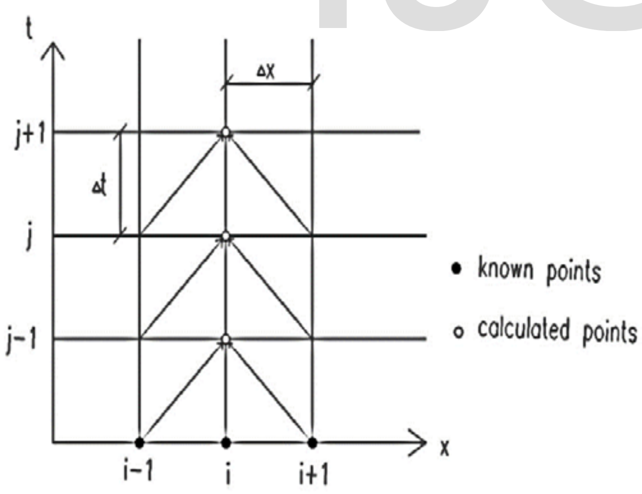

Fig.1. Finite difference grids used in explicit methods

$$\frac{\partial f}{\partial x} = \frac{f_{i+1}^{j} - f_{i-1}^{j}}{2\Delta x} \quad (5)$$

$$\frac{\partial f}{\partial t} = \frac{f_{i}^{j+1} - f^{*}}{\Delta t} \quad (6)$$

$$f^{*} = \frac{1}{2}\left(f_{i-1}^{j} - f_{i+1}^{j}\right) \quad (7)$$

$$D^{*} = \frac{1}{2}\left(D_{i-1}^{j} - D_{i+1}^{j}\right) \quad (8)$$

$$s_f^{*} = \frac{1}{2}\left(s_{f\,i-1}^{\,j} - s_{f\,i+1}^{\,j}\right) \quad (9)$$

Substitution of these expressions in Eq. (1) and Eq. (2) yields:

$$y_i^{j+1} = \frac{1}{2}(y_{i-1}^{j} + y_{i+1}^{j}) - \frac{1}{2}\frac{\Delta t}{\Delta x} D_i^{*}(v_{i+1}^{j} - v_{i-1}^{j}) - \frac{1}{2}\frac{\Delta t}{\Delta x} v_i^{*}(y_{i+1}^{j} + y_{i-1}^{j}) \quad (10)$$

$$v_i^{j+1} = \frac{1}{2}(v_{i-1}^{j} + v_{i+1}^{j}) - \frac{1}{2}\frac{\Delta t}{\Delta x} g\,(y_{i+1}^{j} - y_{i-1}^{j}) - \frac{1}{2}\frac{\Delta t}{\Delta x} v_i^{*}(v_{i+1}^{j} - v_{i-1}^{j}) + g\,\Delta t\,(s_0 - s_f^{*}) \quad (11)$$

3.2 MacCormack Scheme

The MacCormack scheme is also an explicit method, but it is a two-step method in which there is predictor part and corrector part [10]. In prediction part, the following approximations are used:

$$\frac{\partial U}{\partial t} = \frac{U_i^{*} - U_i^{j}}{\Delta t} \quad (12)$$

$$\frac{\partial F}{\partial x} = \frac{F_i^{j} - F_{i-1}^{j}}{\Delta x} \quad (13)$$

Substitution of the approximations mentioned in matrix form of governing equations leads to:
The predictor part:

$$U_i^{*} = U_i^{j} - \frac{\Delta t}{\Delta x}(F_i^{j} - F_{i-1}^{j}) - s_i^{j}\Delta t \quad (14)$$

Where U_i^{*} gives values of A^{*} and Q^{*}, from which we can get values of y and v in which subscript "*" refers to variables which are calculated during the predictor part. After that, these calculated values are used in the corrector part to compute F^{*} and S^{*}.

The corrector part:

$$\frac{\partial U}{\partial t} = \frac{U_i^{**} - U_i^{j}}{\Delta t} \quad (15)$$

$$\frac{\partial F}{\partial x} = \frac{F_{i+1}^{*} - F_i^{*}}{\Delta x} \quad (16)$$

Substitution of these finite differences and $S = S^{*}$ into Eq. 3 leads to :

$$U_i^{**} = U_i^{j} - \frac{\Delta t}{\Delta x}(F_{i+1}^{*} - F_i^{*}) - s_i^{*}\Delta t \quad (17)$$

Where subscripts " $**$ " refers to variables computed during the corrector part. Then the values of U_i at the time step J+1 can be calculated by:

$$U_i^{j+1} = \frac{1}{2}\left(U_i^{*} + U_i^{**}\right) \tag{18}$$

The main disadvantage of all explicit schemes generally is the stability condition known as the courant condition, c_n. The courant number depends upon the values of time interval Δt and the space interval Δx which may require very small values of Δt and Δx. The grid size will be calculated from the condition that the courant number must be less than unity. Courant number is the ratio between actual wave velocity and the numerical wave velocity so c_n is given by:

$$C_n = \frac{v \pm c}{\frac{\Delta x}{\Delta t}} \tag{19}$$

This stability condition may be a complicated process as the courant number should be applied at each time step to get the updated values of the celerity of the wave and the water velocity which depend on the value of variable water stage [6].

Equations of any of the two explicit methods may be used for estimating the flow variables at the interior grid points at each time step. Contrarily, those equations can't be applied to calculate the boundary parameters because there are no grid points outside the flow domain. The characteristics method can be applied to get the flow parameters at the boundaries where the positive characteristic equation may be solved with the downstream boundary condition at the same time, while the negative characteristic equation may be solved with the upstream boundary condition. The end condition may specify time variation of water depth, velocity (or discharge), or a function combining both of them [4, 7].

3.3. Verwey's Variant of Preissmann Implicit Scheme

The main advantage of solving open channel flow problems using implicit schemes generally is the ability to use large time steps without any stability distractions as it may be stable for any condition. Preissmann implicit scheme is applied mostly for solving open channel flow among all implicit techniques.

A scheme of the Preissmann type was modified by Verwey [1,5], who used a different approximation for some terms, as follows:

$$\frac{\partial}{\partial x}\left(\frac{Q^2}{A}\right) = \frac{1}{\Delta x}\left(\frac{Q_{i+1}^{j} Q_{i+1}^{j+1}}{A_{i+1}^{j+0.5}} - \frac{Q_i^{j} Q_i^{j+1}}{A_i^{j+0.5}}\right) \tag{20}$$

$$Q\left|Q\right|K^2 = 0.5\left(\frac{\left|Q_{i+1}^{j}\right| Q_{i+1}^{j+1}}{\left(K_{i+1}^{j+0.5}\right)^2} - \frac{\left|Q_i^{j}\right| Q_i^{j+1}}{\left(K_i^{j+0.5}\right)^2}\right) \tag{21}$$

$$b = 0.5(b_{i+1}^{j+0.5} + b_i^{j+0.5}), A = 0.5(A_{i+1}^{j+0.5} + A_i^{j+0.5}) \tag{22}$$

The superscripts j+0.5 means that the function is computed between two time levels jΔt and (j+1)Δt. Substituting these approximations into equations (1) and (2) leads to:

$$0.5(b_{i+1}^{j+0.5} + b_i^{j+0.5})\left(\frac{y_{i+1}^{j+1} - y_{i+1}^{j}}{2\Delta t} + \frac{y_i^{j+1} - y_i^{j}}{2\Delta t}\right)$$
$$+0.5\left(\frac{Q_{i+1}^{j+1} - Q_i^{j+1}}{\Delta x} + \frac{Q_{i+1}^{j} - Q_i^{j}}{\Delta x}\right) = 0 \tag{23}$$

$$\left(\frac{Q_{i+1}^{j+1} - Q_{i+1}^{j}}{2\Delta t} + \frac{Q_i^{j+1} - Q_i^{j}}{2\Delta t}\right) + \frac{1}{\Delta x}\left(\frac{Q_{i+1}^{j} Q_{i+1}^{j+1}}{A_{i+1}^{j+0.5}} - \frac{Q_i^{j} Q_i^{j+1}}{A_i^{j+0.5}}\right)$$
$$+\frac{g}{2}\left(A_{i+1}^{j+0.5} + A_i^{j+0.5}\right)\left(\frac{y_{i+1}^{j+1} - Q_i^{j+1}}{2\Delta x} + \frac{y_{i+1}^{j} - y_i^{j}}{2\Delta x}\right)$$
$$+\frac{g}{2}\left(A_{i+1}^{j+0.5} + A_i^{j+0.5}\right)\left(\frac{\left|Q_{i+1}^{j}\right| Q_{i+1}^{j+1}}{2\left(K_{i+1}^{j+0.5}\right)^2} + \frac{\left|Q_i^{j}\right| Q_i^{j+1}}{2\left(K_i^{j+0.5}\right)^2}\right)$$
$$-\frac{g}{2}\left(A_{i+1}^{j+0.5} + A_i^{j+0.5}\right)s_0 = 0 \tag{24}$$

The computations begin by setting

$A_i^{j+0.5} = A_i^{j}, b_i^{j+0.5} = b_i^{j}, K_i^{j+0.5} = K_i^{j}$ .The resulting system of linear equations in Q_i^{j+1}, y_i^{j+1} , i=1, 2, 3, --------, J is solved to give a first approximation to these Q_i^{j+1}, y_i^{j+1} values, and a second approximation to the coefficients,

$$b_i^{j+0.5} = 0.5(b_i^{j+1} + b_i^{j}), A_i^{j+0.5} = 0.5(A_i^{j+1} + A_i^{j}),$$
$$K_i^{j+0.5} = 0.5(K_i^{j+1} + K_i^{j}) \tag{25}$$

The second resolution of the linear system leads to the approximation of the unknowns and so on. In this method, the discharge and water depth are computed directly at the same grid points and for that, there is no need to interpolate between points as in Abbott-Lonescu scheme. Also, there is no convergence problem as in the Preissmann scheme [5].

4 INVERSE EXPLICIT METHOD

Several models which are developed to solve operation type problems were based on the simulation techniques since they can describe the complete dynamics of unsteady flow [8]. Inverse explicit scheme is based on discretization of Preissmann implicit scheme. Water depth and discharge at the downstream are two boundary conditions must be known at the time of solution in order to simulate using this method [6].

A set of equations is solved to get the unknown parameters at every time step. Considering the time level 'J' as the final condition, as shown in Fig. 2, and knowing Q_i and y_i , between any two time levels at the downstream section, the discharge and water depth at the previous time level 'J-1' can be computed by proceeding time backward followed by backward in space[11]. In this approach, the discretization of Preissmann scheme is written as:

$$\frac{\partial f}{\partial t} = \phi\frac{f_{i-1}^{j} - f_{i-1}^{j-1}}{\Delta t} + \left(1-\phi\right)\frac{f_i^{j} - f_i^{j-1}}{\Delta t} \tag{26}$$

$$\frac{\partial f}{\partial x}=\theta\frac{f_i^{\,j-1}-f_{i-1}^{\,j-1}}{\Delta x}+\left(1-\theta\right)\frac{f_i^{\,j}-f_{i-1}^{\,j}}{\Delta x} \tag{27}$$

The solution begins at the top corner from the right side of the time-distance plane, Fig 2. Applying the finite difference equations gives a system of two equations in which there are two unknown parameters which can be calculated by solving those two equations. The solution obtained cell by cell, starting from time, followed by space. Introducing inverse explicit scheme into equations (1) and (2) yields the following equations for every two neighboring grid points [12].

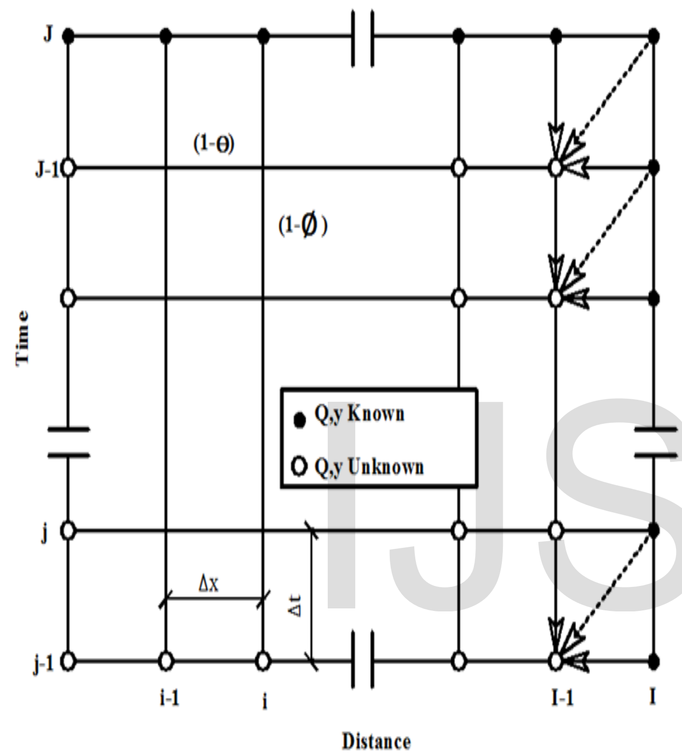

Fig.2. Inverse explicit scheme computational grids

$$P_1Q_{i-1}+Q_1Q_i+R_1y_{i-1}+S_1y_i+T_1=0 \tag{28}$$

$$P_2Q_{i-1}+Q_2Q_i+R_2y_{i-1}+S_2y_i+T_2=0 \tag{29}$$

Where:
y_i & Q_i = water depth and discharge from time level J to (J-1) at grid point I (KNOWNS).
y_{i-1} & Q_{i-1} = water depth and discharge at grid point (I-1) (UNKNOWNS).
P_1, P_2, Q_1, Q_2, R_1, R_2, S_1, S_2, T_1, T_2 are coefficients which can be computed by knowing the values at time level J by applying inverse explicit equations to Saint-Venant equations i.e. Eq. (4) in Eq. (1) and Eq. (2), the discretized forms are:

$$\frac{1}{b}\Big(\frac{\theta\left(Q_i^{j-1}-Q_{i-1}^{j-1}\right)}{\Delta x}+\frac{(1-\theta)\left(Q_i^{j}-Q_{i-1}^{j}\right)}{\Delta x}$$
$$+\frac{\phi}{\Delta t}\left(y_{i-1}^{j}-y_{i-1}^{j-1}\right)+\frac{(1-\phi)}{\Delta t}\left(y_i^{j}-y_i^{j-1}\right)=0 \tag{30}$$

$$\frac{\phi}{\Delta t}\left(Q_{i-1}^{j}-Q_{i-1}^{j-1}\right)+\frac{(1-\phi)}{\Delta t}\left(Q_i^{j}-Q_i^{j-1}\right)$$
$$+\left(\frac{\theta}{\Delta x}\left(\frac{Q^2}{A}{}_i^{j-1}-\frac{Q^2}{A}{}_{i-1}^{j-1}\right)+\frac{(1-\theta)}{\Delta x}\left(\frac{Q^2}{A}{}_i^{j}-\frac{Q^2}{A}{}_{i-1}^{j}\right)\right)$$
$$+gA\Big(\frac{\theta}{\Delta x}\left(y_i^{j-1}-y_{i-1}^{j-1}\right)+\frac{(1-\theta)}{\Delta x}\left(y_i^{j}-y_i^{j-1}\right)-s_0+s_f\Big)=0 \tag{31}$$

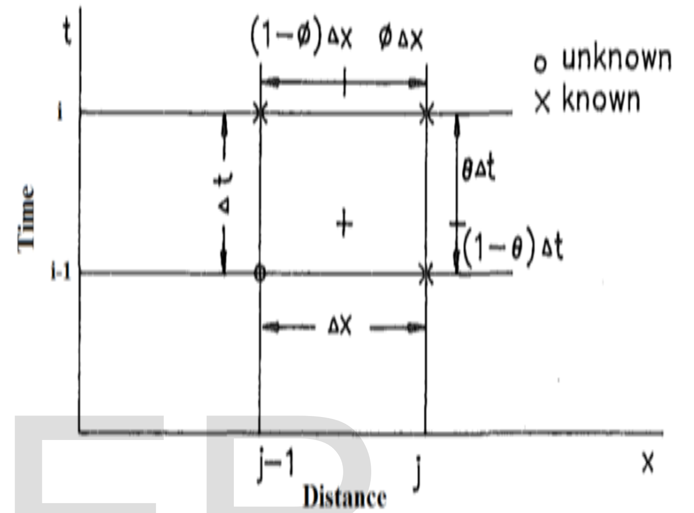

Fig.3. Computational Grid for inverse explicit scheme.

Comparing the coefficients, we can get the value of P_1, Q_1, R_1, S_1, T_1, P_2, Q_2, R_2, S_2 and T_2. Knowing Q_i and y_i between any two time levels at the last section of the channel, one can apply equations (28) and (29) for the last two sections i-1 and i in (Fig. 2), and solve Q_{i-1} and y_{i-1} explicitly:

$$Q_{i-1}=\frac{R_1\left(Q_2Q_i+S_2y_i+T_2\right)-R_2\left(Q_1Q_i+S_1y_i+T_1\right)}{P_1R_2-P_2R_1} \tag{32}$$

$$y_{i-1}=\frac{P_1\left(Q_2Q_i+S_2y_i+T_2\right)-P_2\left(Q_1Q_i+S_1y_i+T_1\right)}{P_2R_1-P_1R_2} \tag{33}$$

5. Description of the First Case Study

The example presented in Liu, et al. 1992 [8] was used to test the results found by inverse explicit finite difference technique. The method was applied to unsteady flow in a channel which was in the shape of a trapezoidal section with 5.0m bottom width and side slopes are 1.5:1. The bottom slope is 0.001, Manning's roughness coefficient is 0.025, the channel length is 2.5 km, and there was a fixed overflow weir which was considered as a downstream outlet condition. At the downstream section, the discharge of the flow increases from a discharge value of 5 m^3/sec to 10 m^3/sec in one hour, there is no change in the discharge value of 10 m^3 /sec in the next two hours, then the flow discharge decreases to 5.0 m^3/sec in the next hour (demand line in Fig. 4.). At the downstream section, there was a fixed weir under a free-flow condition which was used to get the relationship between the discharge and the water depth. As the initial conditions, the water depth and discharge at the upstream section, Figs. 4 and 5, were computed using the specified water depth and discharge at the downstream section. The calculated hydrograph at the upstream section was then used to simulate the channel flow with the routing finite difference schemes. The calculated downstream hydrographs reasonably reproduced the demand downstream, Figs. 4 and 5.

In the routing problem, the upstream hydrograph calculated using inverse explicit scheme ($\Delta x = 100m, \Delta t = 200\sec, \phi = 0.5, \theta = 1$) is used as upstream boundary condition to get the downstream hydrograph using different finite difference schemes. Lax diffusive scheme and MacCormack scheme are used as explicit schemes and Verwey Veriant's of Preissmann scheme is used as an implicit scheme. Comparison between the results found by the three methods has been done. Implicit schemes are distinguished from explicit schemes as they are unconditionally stable. So, while using the Preissmann implicit scheme, there are no constraints or conditions in choosing values of space interval and time interval. On the other hand while dealing with both explicit schemes, courant condition must be achieved. This courant condition requires small values of space interval and time interval in order to be achieved and get acceptable results. Consequently, in the present case study the same space interval and time interval are used in both explicit methods ($\Delta x = 30m, \Delta t = 6\sec$) to make a fair comparison between them.

It can be noticed from table (1), figs. 4 and 5 that the three methods gave acceptable results with a small percentage of error and any of them can be used for solving the routing problems. It is also found that the inverse explicit method is stable and reproduced downstream flow hydrographs very close to the demand outflow.

TABLE 1

COMPARISON BETWEEN DIFFERENT ROUTING METHODS

Comparison method	Mean relative error %		Mean absolute error	
	Q	y	Q	Y
Lax diffusive scheme	1.88	.48	0.11	0.008
MacCormack scheme	1.03	.66	0.099	0.015
Preissmann scheme	1.34	0.40	0.09	0.007

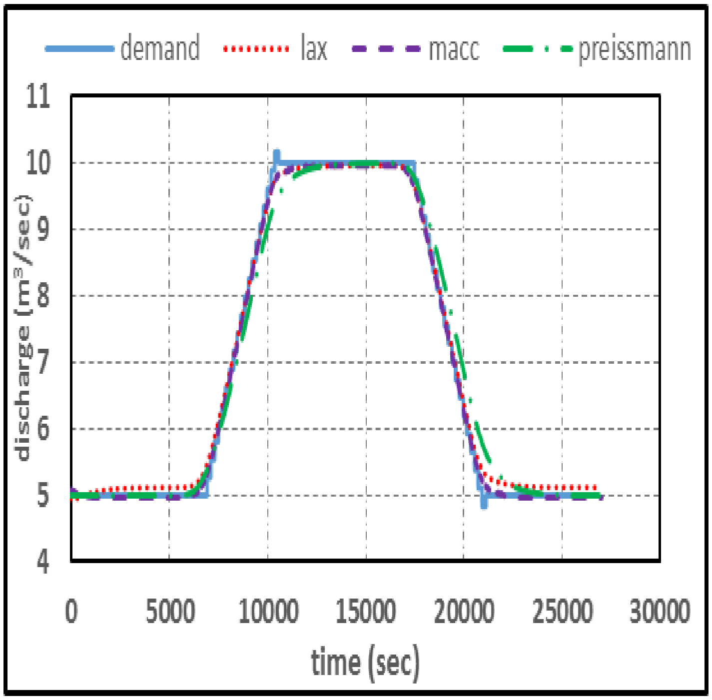

Fig.4 Comparison between downstream discharge hydrographs using different finite difference schemes.

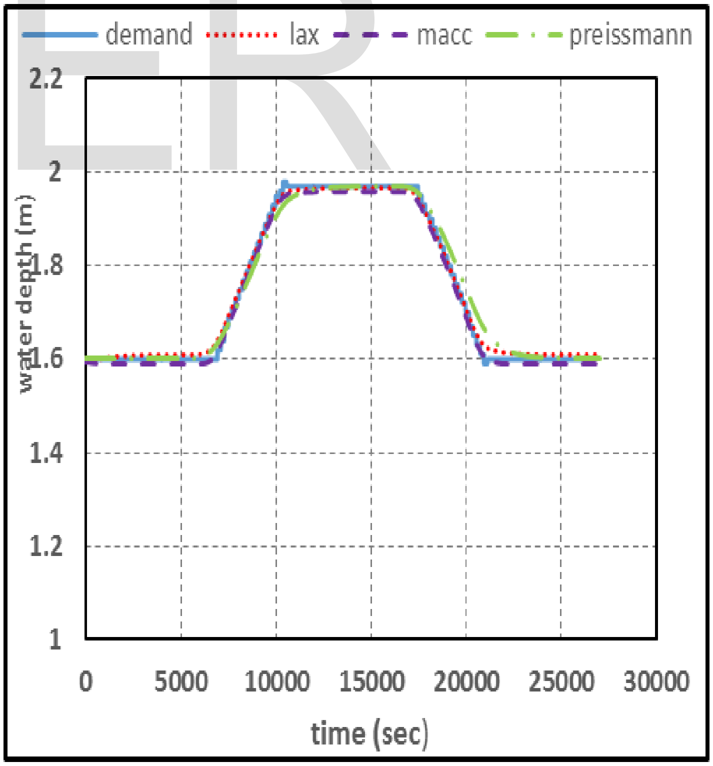

Fig. 5 Comparison between downstream water depth hydrographs using different finite difference schemes.

6. Description of Second Case Study

A reach of 18km length from Al-Mansouria canal was used to test the results found by inverse explicit finite difference technique as an example of non-prismatic section with an irregular shape. The cross section spacing is 2.0 km, the channel bed slope is 4.5 cm/ km [11]. The value of Manning's roughness coefficient is 0.025. At the downstream of the channel, the discharge increases from 50 m^3/sec to 55 m^3/sec in a time period of one hour, it remains constant at 55 m^3/sec for two hours, then decreases to 50.0 m^3/sec in one hour. The manning equation was applied to obtain the water depth at the downstream end section. The discharge and water depth at the upstream intake were computed using the specified discharge and water depth at the downstream end section as the initial condition [13].As mathematical solution of Saint-Venanat equations requires that the wetted perimeter 'P', wetted cross sectional area 'A', top width 'T' must be known as a function of the water depth 'y'. Those functions may be calculated easily using the given field data shown in Shammaa 1988 [11]. At the first time step, both water depth and discharge at all points of the channel must be specified. Two boundary conditions are needed for the downstream section. This condition can be achieved by giving the discharge hydrograph and the corresponding water depth hydrograph at the downstream sections. In our case, the initial condition is given as 50.0 m^3/s for discharge with the corresponding water depth [13].

The inverse explicit scheme was tested using different space intervals (Δx), different time intervals (Δt), different weighting coefficients (Φ) and θ.

The calculated discharge hydrograph using inverse explicit method for space intervals (Δx) = 1000m, 2000m, and 3000m with time interval (Δt) =1800sec, weighting coefficients (Φ) =.5 and $\theta = 1$ are shown in Fig.6. It is clearly seen from the figure that there was no obvious change between the upstream produced by using different space intervals and all of them gave downstream hydrograph very close to the demand. The accuracy of each space interval (Δx) = (1000m, 2000m, 3000m) is (98.14%, 98.15%, 98.17%) respectively.

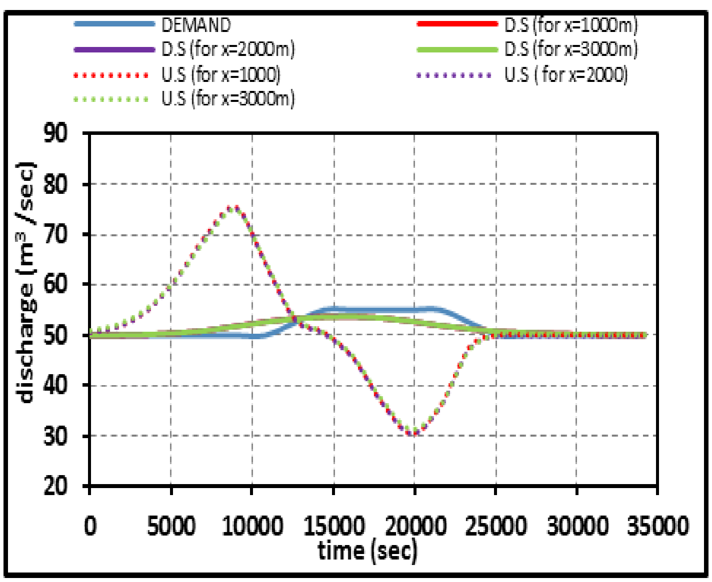

Fig. 6 The computed discharge hydrographs using different distance intervals in inverse explicit method.

The computed discharge hydrograph using inverse explicit method for time steps (Δt) = 1000sec, 1800sec, and 3000sec with space interval (Δx) =1000m, weighting coefficients (Φ) =.5 and $\theta = 1$ are shown in Fig.7. It is clearly seen from the figure that small time intervals may show fluctuation in the upstream section. This may lead to using a larger time intervals to show stable downstream results with an acceptable percentage of error. The accuracy of each time interval ($\Delta t = 1800, 3000\,\text{sec}$) is (98.14%, 97.66%) respectively.

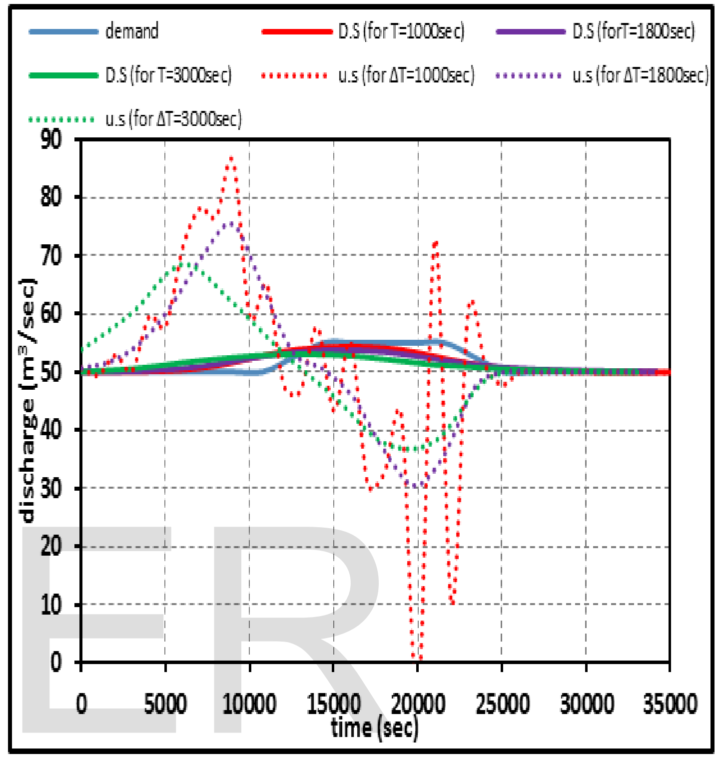

Fig. 7 The computed discharge hydrograph using different time intervals.

The computed discharge hydrograph using inverse explicit method for weighting coefficient (Φ) = .5, .7 and .99 with space interval (Δx) = 1000m, time interval (Δt) =1800sec and weighting coefficient $\theta = 1$ are shown in fig 8. It is clearly seen from the figure that the weighting coefficients (Φ) has a small effect on the computed upstream hydrographs and reproduces approximately the same downstream hydrographs. The accuracy of each weighting coefficient (ϕ =.5, .7, 1) are (98.15%, 97.62%, 97.72%) respectively.

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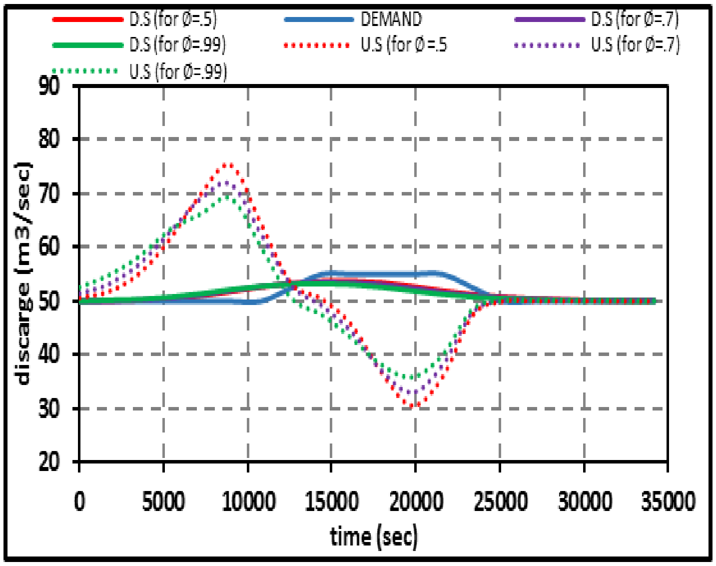

Fig. 8 The computed discharge hydrograph using different weighting coefficient (Φ).

The computed discharge hydrograph using inverse explicit method for weighting coefficient(θ) = .8, .9 and 1 with space interval (Δx) =1000m, time interval (Δt) =1800 sec and weighting coefficient $\phi = 1$ are shown in to Fig. 9. It is clearly seen from the figure that the oscillation is damped when weighting coefficient (θ) is larger than .8 with accurate results and small oscillation. The accuracy of each weighting coefficient (θ=1,0.9,0.8) are (98.15%,98.38%, 98.65) respectively.

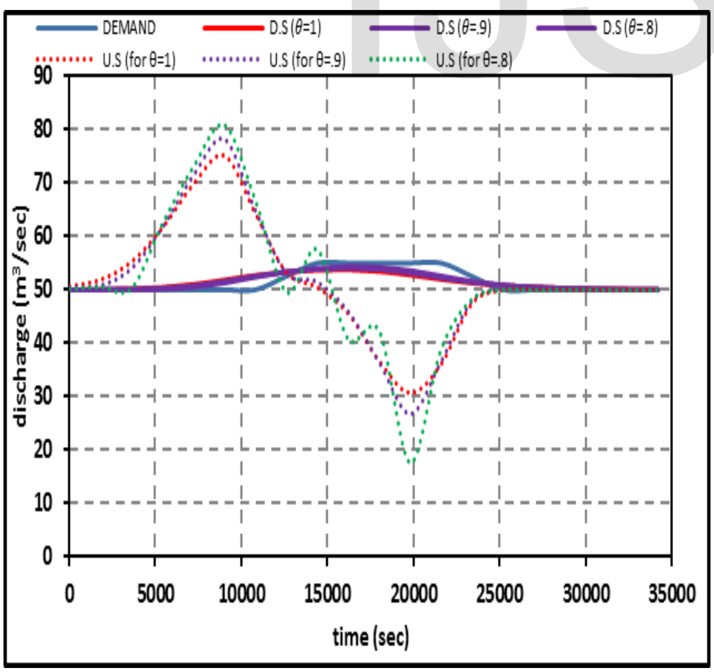

Fig.9. The computed discharge hydrograph using different weighting coefficient (θ).

7. Description of Third Case Study

A channel diverted into two branches was used to test the results found by inverse explicit finite difference technique (Fig.10.). The main canal and both branches have a trapezoidal section with a bottom width 5.0m and side slopes are 1.5:1. The bottom slope is 0.001, Manning's coefficient is 0.025, the channel length is 2.5 km, and there was a fixed overflow weir at the end of every branch with free flow condition. At downstream outlet for the two branches, the discharge increases from 2.5 m^3/sec to 5 m^3/sec in one hour, it remains constant at 5 m^3 /sec for the next two hours, then decreases to 2.5 m^3/sec in one hour (demand line in Fig.11.). The relationship between discharge and water depth was used to calculate the water depth at the end section. The discharges at the upstream of the two branches, which are shown in Fig.11, were computed using the specified discharge and water depth at the downstream end section as the initial conditions. The continuity equation was used at the connecting point to be used as a downstream boundary condition to get the discharge at the upstream section of the main canal. Then the process goes in its forward direction again to verify the results obtained. Obtained upstream discharge hydrograph was used as the upstream boundary condition, to simulate the flow in the channel with the routing implicit finite difference scheme. Then the downstream gained in the main canal is used to get the upstream boundary condition in both branches. The computed downstream hydrographs reasonably reproduced the prescribed demand at each branch, Figs. 12.

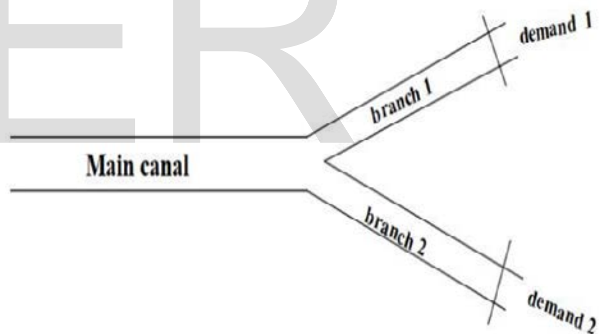

Fig.10. Canal diverted into two branches.

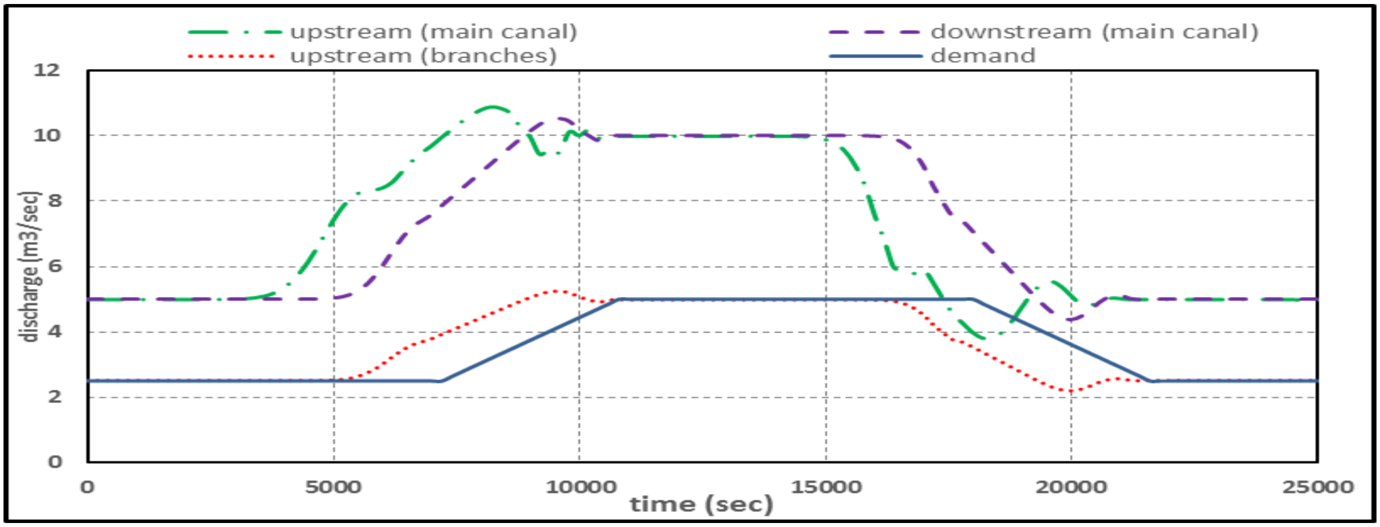

Fig. 11. The computed discharge hydrograph using inverse explicit method for the two typical branches.

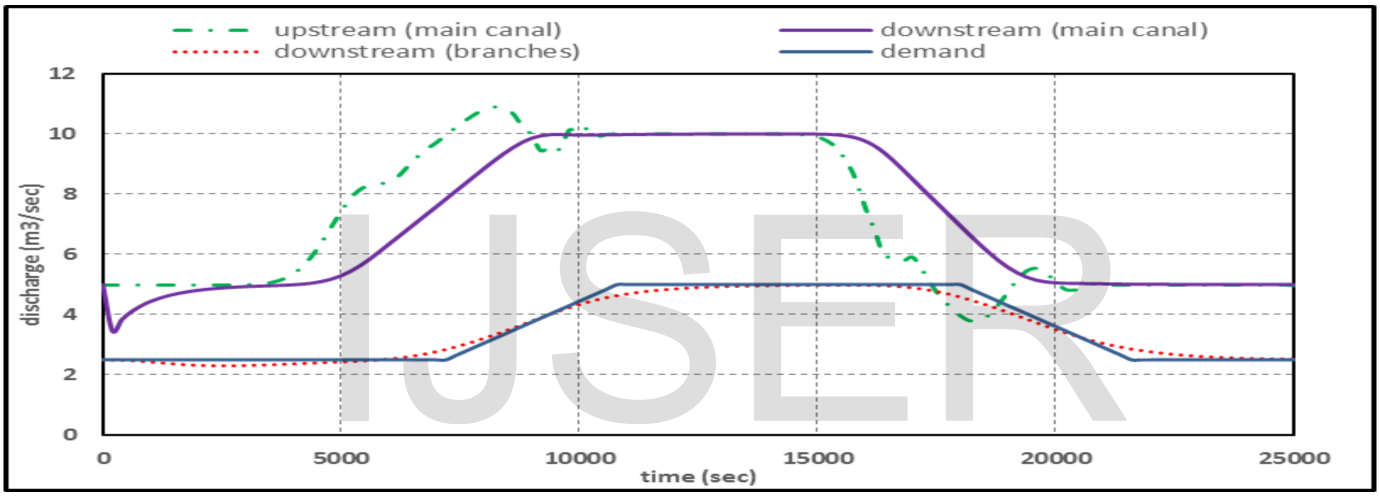

Fig. 12. The computed discharge hydrograph using routing implicit method for the two typical branches.

Finally the inverse explicit scheme was tested using the same case study but with changing the length of one branch to be 5.0 km instead of 2.5 km as shown in figures 13, 14.

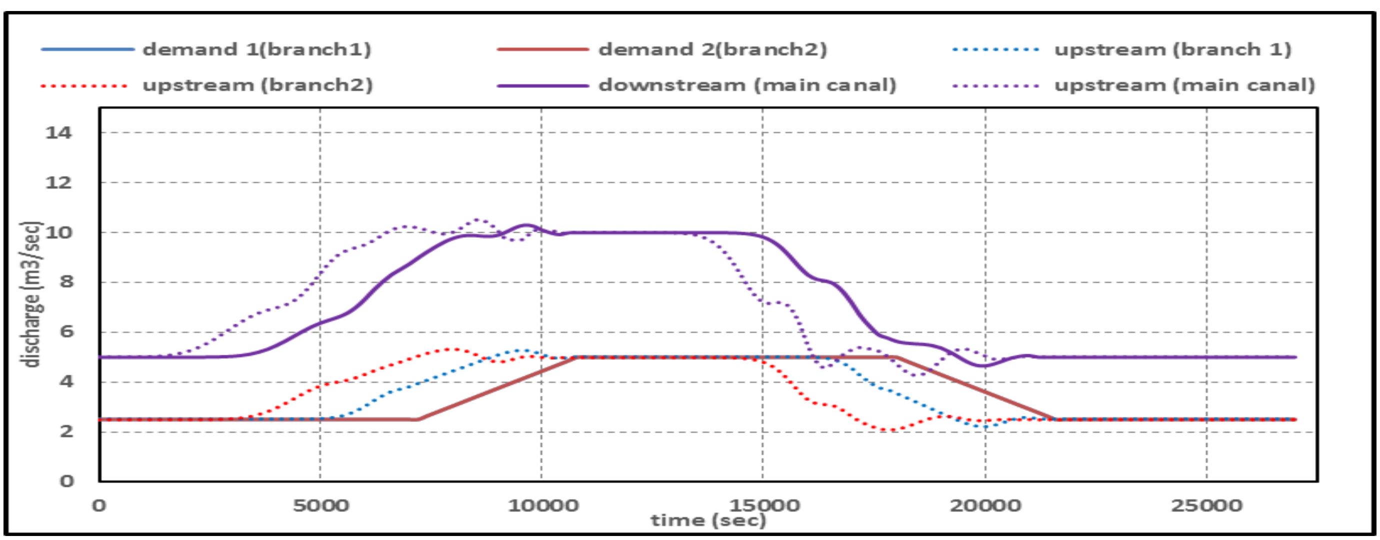

Fig. 13. The calculated hydrograph of discharge using inverse explicit method for different lengths of branch canals.

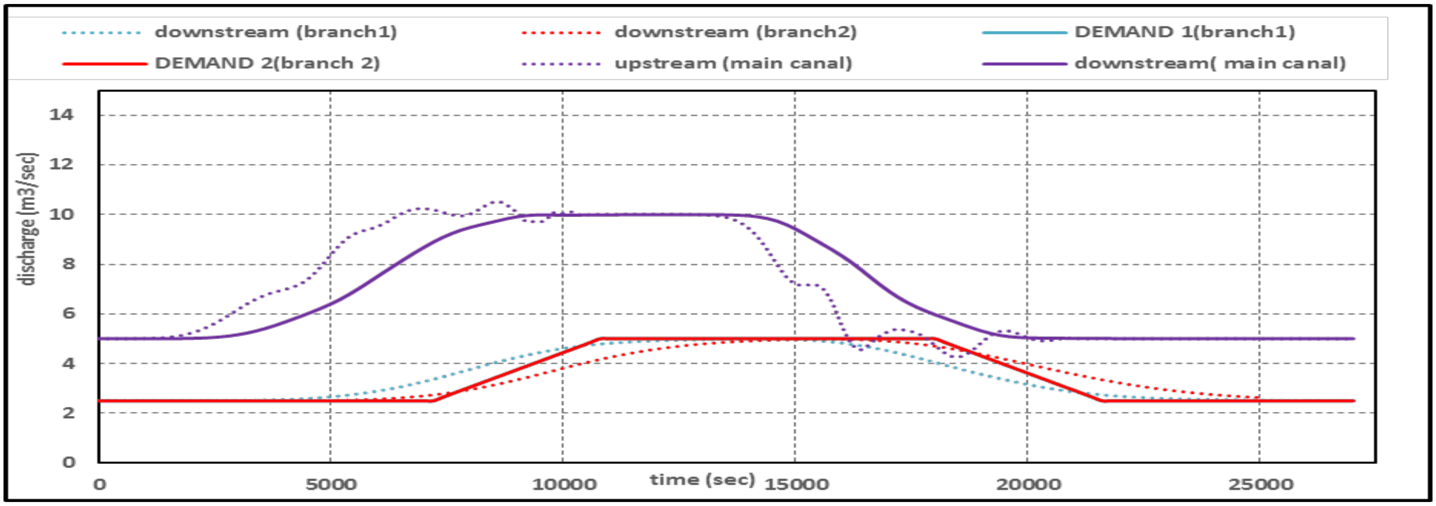

Fig.14.The computed discharge hydrographs using routing implicit method for different lengths of branch canals.

It can be noticed from the two case studies that the computed downstream hydrographs reasonably reproduced the prescribed demand at each channel with a reasonable error (the first case study with mean relative error =3.55 % at each branch as they are equal and the second case study with mean relative error= 7.01%, 6.92% at the two branches). As it is shown from the percentage of error that all of them may be acceptable, but any change between the two branches reproduces more error in the results.

8. Conclusion

Unsteady flow problems in open channels can be classified as routing and operation type problems, according to the objectives of the study. Both of routing problems and operation type problems were discussed. Lax diffusive explicit scheme, MacCormack explicit scheme and Verwey Verian't of Preissmann implicit scheme were used for solving the routing problems. Inverse explicit method which is based on Preissmann scheme was used for solving the operation type problems.

Firstly, a comparison between the three routing methods has been done. It was found that the three methods gave acceptable results with a small percentage of error and any of them can be used for solving the routing problems. It was found also that the inverse explicit method is stable and reproduced downstream flow hydrographs very close to the predefined outflow.

The inverse explicit scheme was tested using the following parameters: the space interval Δx , the time interval Δt , the weighting coefficient ϕ and the weighting coefficient θ for a non-prismatic area of Al-Mansouria canal between Snayet regulator and Bhr Tanah canal.

Firstly, while comparing between different space intervals, there was no obvious change between the downstream produced by all of them and all of them are very close to the demand downstream.

Secondly, while using different time intervals, it was found that small time intervals may show fluctuation at the upstream boundary section. This may lead to using a larger time intervals to show stable downstream results with an acceptable percentage of error.

Thirdly, concerning the effect of using different weighting coefficient ϕ from (0.5 to 1.0), it was found that although using smaller values of ϕ gave the least percentage of error, it showed more fluctuation in the calculated upstream hydrograph. Contrarily, using larger values of ϕ showed less fluctuation in the upstram and more error in the calculated downstream.

Finally, while using different weighting coefficient θ, it was concluded that the fluctuation of the calculated upstream hydrographs were damped when θ increased from 0.8 to 1.0. All used values of θ gave very close results to the predefined hydrograph.

The performance of the inverse explicit finite difference scheme was tested using unsteady flow in a channel diverted into two branches. The same case study was solved two times; first with the same specifications for the two branches, then with difference in length of the two branches. It was found that at each branch of the channel, the computed downstream hydrographs reasonably reproduced the demand hydrograph with a reasonable error (the first case with mean error =3.55 % at each branch as they are equal, the second case study with mean error= 7.01%, 6.92% at the two branches). As it shown from the percentages that all of them may be acceptable, but any change between the two branches may reproduce more errors.

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NOTATIONS

A = wetted cross-sectional area
g = gravitational acceleration
VA =Q= discharge (through A)
y = depth of flow
t = time
x = space
S_0 =bottom slope of the channel
S_f =friction slope
$Ay^{/}$ =moment of flow area about the free surface.
T = wetted top width
f = general function which may be depth of flow, velocity of flow or discharge of flow
i = cross-section index
j = time-level index
n=Manning's coefficient
Δt = time interval
Δx = space interval
φ = a weighting coefficient for distributing terms in space
θ = a weighting coefficient for distributing terms in time.